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Old inflation in string theory

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Abstract: We propose a stringy version of the old inflation scenario which does not require any slow-roll inflaton potential and is based on a specific example of string compactification with warped metric. Our set-up admits the presence of anti-$D3$-branes in the deep infrared region of the metric and a false vacuum state with positive vacuum energy density. The latter is responsible for the accelerated period of inflation. The false vacuum exists only if the number of anti-$D3$-branes is smaller than a critical number and the graceful exit from inflation is attained if a number of anti-$D3$-branes travels from the ultraviolet towards the infrared region. The cosmological curvature perturbation is generated through the curvaton mechanism.

Keywords: D-branes, Cosmology of Theories beyond the SM.

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Contents

1. Introduction

Inflation has become the standard paradigm for explaining the homogeneity and the isotropy of our observed Universe [1]. At some primordial epoch, the Universe is trapped in some false vacuum and the corresponding vacuum energy gives rise to an exponential growth of the scale factor. During this phase a small, smooth region of size of the order of the Hubble radius grew so large that it easily encompasses the comoving volume of the entire presently observed Universe and one can understand why the observed Universe is homogeneous and isotropic to such high accuracy. Guth's original idea [2] was that the end of inflation could be initiated by the tunneling of the false vacuum into the true vacuum during a first-order phase transition. However, it was shown that the created bubbles of true vacuum would not percolate to give rise to the primordial plasma of relativistic degrees of freedom [3]. This drawback is solved in slow-roll models of inflation [1] where a scalar field, the inflaton, slowly rolls down along its potential. The latter has to be very flat in order to achieve a sufficiently long period of inflation which is terminated when the slow-roll conditions are violated. The Universe enters subsequently into a period of matter-domination during which the energy density is dominated by the coherent oscillations of the inflaton field around the bottom of its potential. Finally reheating takes place when the inflaton decays and its decay products thermalize.

While building up successful slow-roll inflationary models requires supersymmetry as a crucial ingredient, the flatness of the potential is spoiled by supergravity corrections, making it very difficult to construct a satisfactory model of inflation firmly rooted in in modern particle theories having supersymmetry as a crucial ingredient. The same difficulty is encountered when dealing with inflationary scenarios in string theory. In brane-world scenarios inspired by string theory a primordial period of inflation is naturally achieved [4] and the role of the inflaton is played by the relative brane position in the bulk of the underlying higher-dimensional theory [5]. In the exact supersymmetric limit the brane position is the Goldstone mode associated to the translation (shift) symmetry and the inflaton potential is flat. The weak brane-brane interaction breaks the translational shift symmetry only slightly, giving rise to a relatively flat inflaton potential and allowing a sufficiently long period of inflation. However, the validity of brane inflation models in string theory depends on the ability to stabilize the compactification volume. This means the effective four-dimensional theory has to fix the volume modulus while keeping the potential for the distance modulus flat. A careful consideration of the closed string moduli reveals that the superpotential stabilization of the compactification volume typically modifies the inflaton potential and renders it too steep for inflation [6]. Avoiding this problem requires some conditions on the superpotential needed for inflation [7, 8].

In this paper we propose a stringy version of the old inflation scenario which does not require any slow-roll inflaton potential and is based on string compatifications with warped metric. Warped factors are quite common in string theory compactifications and arise, for example, in the vicinity of D-branes sources. Similarly, string theory has antisymmetric forms whose fluxes in the internal directions of the compactification typically introduce warping. In this paper we focus on the Klebanov-Strassler (KS) solution [9], compactified as suggested in [10], which consists in a non-trivial warped geometry with background values for some of the antisymmetric forms of type-IIB supergravity. The KS model may be thought as the stringy realization of the Randall-Sundrum model (RSI) [11], where the Infra-Red (IR) brane has been effectively regularized by an IR geometry, and has been recently used to embed the Standard Model on anti-$D3$-branes [12].

From the inflationary point of view, the basic property of the set-up considered in this paper is that it admits in the deep IR region of the metric the presence of p anti-$D3$-branes. These anti-branes generate a positive vacuum energy density as in [13]. We consider a specific scenario studied in [14] where the anti-branes form a metastable bound state. For sufficiently small values of p, the system sits indeed on a false vacuum state with positive vacuum energy density. The latter is responsible for the accelerated period of inflation. In terms of the four-dimensional effective description, the inflaton may be identified with a four-dimensional scalar field parameterizing the angular position ψ of the anti-$D3$-branes in the internal directions. The curvature of the potential around the minimum is much larger than H_*^2, being H_* the value of the Hubble rate during inflation, and slow-roll conditions are violated. Under these circumstances, inflation would last for ever. The key point is that, as shown in [14], the false vacuum for the potential $V(\psi)$ exists only if the number of anti-$D3$-branes is smaller than a critical number. If a sufficient number of anti-$D3$-branes travels from the Ultra-Violet (UV) towards the IR region, thus

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increasing the value of p, inflation stops as soon as p becomes larger than the critical value. At this point, the curvature around the false vacuum becomes negative, the system rolls down the supersymmetric vacuum and the graceful exit from inflation is attained.

The nice feature about this scenario is that it may be considered intrinsically of stringy nature. Indeed, the false vacuum energy may assume only discrete values. Furthermore, even though the dynamics of each wandering anti-$D3$-brane may be described in terms of the effective four-dimensional field theory by means of a scalar field parameterizing the distance between the wandering anti-$D3$-brane and the stack of p anti-$D3$-branes in the IR, the corresponding effective inflationary model would contain a large number of such scalars whose origin might be considered obscure if observed from a purely four-dimensional observer [15]. In this paper we consider a specific set-up that has been already analyzed in literature [14], however one could envisage other stringy frameworks sharing the same properties described here.

The last ingredient we have to account for in order to render our inflationary scenario attractive is to explain the origin of cosmological perturbations. It is now clear that structure in the Universe comes primarily from an almost scale-invariant superhorizon curvature perturbation. This perturbation originates presumably from the vacuum fluctuation, during the almost-exponential inflation, of some field with mass much less than the Hubble parameter H_*. Indeed, every such field acquires a nearly scale-invariant classical perturbation. In our scenario, the inflaton field mass is not light compare to H_* and its fluctuations are therefore highly suppressed. However, its has been recently proposed that the field responsible for the observed cosmological perturbations is some 'curvaton' 'field different from the inflaton [16]. During inflation, the curvaton energy density is negligible and isocurvature perturbations with a flat spectrum are produced in the curvaton field. After the end of inflation, the curvaton field oscillates during some radiation-dominated era, causing its energy density to grow and thereby converting the initial isocurvature into curvature perturbation. This scenario liberates the inflaton from the responsibility of generating the cosmological curvature perturbation and therefore avoids slow-roll conditions. We will show that in our stringy version of the old inflation it is possible to find scalar fields which have all the necessary properties to play the role of the curvaton.

The paper is organized as follows. In section 2 we describe our stringy set-up deferring the technicalities to the appendices A, B, C. In section 3 we describe the properties of the inflationary stage and the production of the cosmological perturbations. Finally, in section 4 we draw our conclusions. The appendices A, B, C provide some of the details about the set-up discussed in section 2.

2. The set-up

We consider a specific example of string compactification with warped metric that has all features for realizing old inflation in string theory. Here we will give a summary of the properties of the model, referring to the appendices A, B, C for a detailed discussion of the compactification and for a derivation of the various formulae.

We are interested in a string compactification where we can insert branes and antibranes at specific locations in the internal directions. An explicit solution where one can actually study the dynamics of the inserted branes is the Klebanov-Strassler (KS) solution [9], compactified as suggested in [10]. The KS solution consists in a non-trivial warped geometry with background values for some of the antisymmetric forms of type-IIB supergravity. In particular, the RR four form $C_{(4)}$ and the (NS-NS and R-R) two forms $B_{(2)}, C_{(2)}$ are turned on. There are indeed N units of flux for $C_{(4)}$ and M units of flux for $C_{(2)}$ along some cycles of the internal geometry. The solution is non compact and one can choose a radial coordinate in the internal directions that plays the familiar role of the fifth dimension in the AdS/CFT correspondence and in the Randall-Sundrum (RS) models. The KS solution can be compactified by gluing at a certain radial cut-off a compact Calabi-Yau manifold that solves the supergravity equations of motion. In the compact model, one also adds an extra flux K for $B_{(2)}$ along one of the Calabi-Yau cycles [10]. Summarizing, the solution is specified by the strings couplings α' and g_s and by three integer fluxes N, M and K. These numbers are constrained by the tadpole cancellation condition (charge conservation). A detailed discussion of these constraints can be found in [10] and it is reviewed in the appendices A, B, C. The internal manifold typically has other non-trivial cycles. If needed, extra parameters can be introduced by turning on fluxes on these cycles.

The KS solution was originally found in the the contest of the AdS/CFT correspondence as the supergravity dual of a confining $N = 1$ gauge theory. It represents the near horizon geometry of a stack of three-branes in a singular geometry in type-IIB string theory. As familiar in the AdS/CFT correspondence, by looking at the near horizon geometry of a system of branes, one obtains a dual description in terms of a supergravity theory. Notice that in the KS supergravity solution there is no explicit source for the branes: the there-branes sources have been replaced by fluxes of the antisymmetric forms along the non trivial cycles of the internal geometry. The reason for this replacement is that D-branes in type-II strings are charged under the RR-forms. The fluxes in the KS solution recall that the background was originally made with (physical and fractional) three-branes charged under $C_{(4)}$ and $C_{(2)}$. However one must remember that the correct relation between branes and fluxes is through a string duality (the AdS/CFT correspondence).

To our purposes, the KS model has two important properties. The first one is that there is a throat region where the metric is of the form

$$ds^2 = h^{-1/2}(r)dx_\mu dx^\mu + h^{1/2}(r)(dr^2 + r^2 ds_{(5)}) \tag{2.1}$$

for $r < r_{UV}$. Here we have chosen a radial coordinate in the compact directions and we have indicated with $ds_{(5)}$ the angular part of the internal metric. For $r > r_{UV}$, (2.1) is glued with a metric that compactifies the coordinates r and contains most of the details of the actual string compactification. The important point here is that the warp factor $h(r)$ is never vanishing and has a minimal value $h(r_0)$. Roughly speaking, the model is a stringy realization of the first Randall-Sundrum model (RSI), where the IR brane has been effectively regularized by an IR geometry. In first approximation, for r sufficiently large, the reader would not make a great mistake in thinking to the RSI model with a Planck

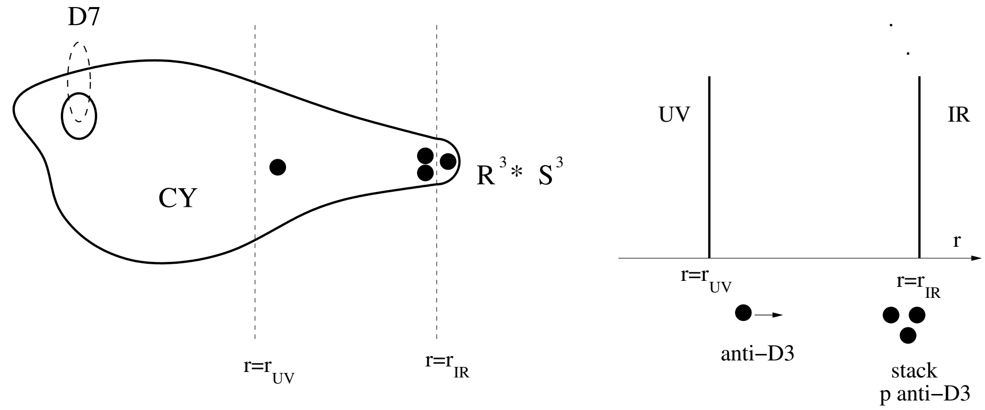

Figure 1: The CY compactification with a throat and its corresponding interpretation in terms of a simplified RSI model. Recall that small r means IR. $D7$ branes that might serve for generating non-perturbative superpotentials are naturally present in the UV region.

brane at r_{UV} and an IR brane at r_0 (see figure 1). To avoid confusions, it is important to stress that, in our coordinates, large r corresponds to the UV region and small r to the IR (in the RS literature one usually considers a fifth coordinate z related to r by $r = e^{-z}$).

In particular, for $r \gg r_0$ (but $r < r_{UV}$) the warp factor is approximately $h(r) = R^4/r^4$ and the metric for the five coordinates (x_μ, r) is Anti-de-Sitter as in the RSI model. In particular, we will write $\min h(r) = R^4/r_0^4$ in the following. The precise relation between parameters identifies [10]

$$\begin{aligned} N &= MK\,, \\ R^4 &= \frac{27}{4}\pi g_s N \alpha'^2\,, \\ \frac{r_0}{R} &\sim e^{-\frac{2\pi K}{3 g_s M}}\,, \\ M_p^2 &= \frac{2\mathcal{V}}{(2\pi)^7 \alpha'^4 g_s^2}\,, \end{aligned} \tag{2.2}$$

where $\mathcal{V}$ is the internal volume and M_p is the four-dimensional Planck mass. Here $\mathcal{V}$ is a modulus of the solution even though the dependence of the warp factor on the volume can be subtle [10].

The second property deals with the IR region of the metric. We are interested in putting p anti-$D3$-branes in the deep IR. They will break supersymmetry and provide the positive vacuum energy necessary for inflation. While the RSI model is not predictive about the fate of the anti-branes in the IR, in the KS model we can analyze their dynamics. It was observed in [14] that the p anti-$D3$-branes form an unstable bound state. If the background were really made of branes, p anti-$D3$-branes would annihilate with the existing $D3$ branes. In the actual background, one should think that the anti-$D3$-branes finally annihilate by transforming into pure flux for $C_{(4)}$, under which they are negatively charged. The fate of the anti-$D3$-brane can be studied using string theory. The mechanism [14], which is reviewed in appendix C, is roughly as follows. It is energetically favorable for the anti $D3$-branes to expand in a spherical shell in the internal directions (see figure 2). The spherical

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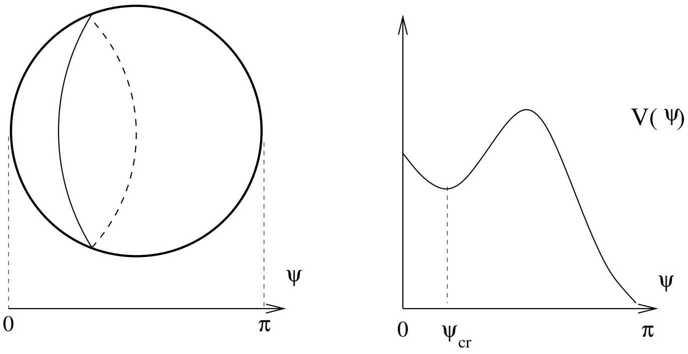

Figure 2: The expanded anti-$D3$-branes and the form of the potential for small values of p/M.

shell is located in the IR at the point r_0 of minimal warp factor. The IR geometry is, in a good approximation, $R^7 \times S^3$ and the branes distribution wraps a two sphere inside S^3.

The set of all $S^2 \subset S^3$ is parameterized by an angle $\psi \in [0, \pi]$, where $\psi = 0$ corresponds to the North pole and $\psi = \pi$ the South pole. The angular position ψ of the branes appears as a scalar in the world-volume action. As shown in [14] the branes feel a potential in ψ. The dynamics of the scalar field ψ can be summarized by the lagrangian

$$L(\psi) = \int d^4x\sqrt{g}\left\{ M_p^2\mathcal{R} - T_3\frac{r_0^4}{R^4}\left[M\left(V_2(\psi)\sqrt{1-\alpha'\frac{R^2}{r_0^2}(\partial\psi)^2} - \frac{1}{2\pi}(2\psi - \sin 2\psi)\right) + p\right]\right\}, \tag{2.3}$$

where $T_3 = \frac{1}{(2\pi)^3 g_s(\alpha')^2}$ is the tension of the anti-$D3$-branes and

$$V_2(\psi) = \frac{1}{\pi}\sqrt{b_0^2\sin^4\psi + \left(\frac{\pi p}{M} - \psi + \frac{\sin 2\psi}{2}\right)^2}, \tag{2.4}$$

with $b_0 \equiv 0.9$.

For sufficiently small p/M, the potential

$$V(\psi) = MT_3\left(\frac{r_0}{R}\right)^4\left[V_2(\psi) - \frac{1}{2\pi}(2\psi - \sin 2\psi) + \frac{p}{M}\right] \tag{2.5}$$

has the form pictured in figure 2. The original configuration of p anti-$D3$-branes can be identified with a (vanishing) spherical shell at $\psi = 0$. The total energy of the configuration is

$$V(\psi_{cr}) \equiv V_0 = 2pT_3\left(\frac{r_0}{R}\right)^4, \tag{2.6}$$

where $(r_0/R)^4$ is due to the red-shift caused by the warped metric and the factor of two is determined by an interaction with the background fluxes explained in [14]. As shown in figure 2, it is energetically favorable for the branes to expand until ψ reaches the local minimum at ψ_{cr}. The configuration is only metastable; the true minimum is at $\psi = \pi$ where the shell is collapsed to a point and the energy of the system vanishes. This means that the anti-branes have disappeared into fluxes: the final state is supersymmetric and of the same form of the KS solution with a small change in the fluxes: $M \to M - p$ and in $K \to K - 1$ [14].

Consider an initial configuration where the bound state of anti-branes is in the false vacuum ψ_{cr}. This provides a vacuum energy $V(\psi_{cr})$ that causes inflation. For small values of p/M, the critical value of $\psi \sim p/M$ is near zero and the vacuum energy of the configuration is approximatively given by (2.6). The mass squared of the fluctuation ψ around the false vacuum can be computed using the lagrangian (2.3) and reads approximatively $m_\psi^2 \sim (1/\alpha')(r_0/R)^2$. With a reasonable choice of parameters, we can easily get $m_\psi^2 \gg H_*^2 = V(\psi_{cr})/M_p^2$, where

$$H_*^2 = \frac{V_0}{3M_p^2} \simeq 2p\left(\frac{r_0}{R}\right)^4 \frac{T_3}{3\,M_p^2}\,, \tag{2.7}$$

is the Hubble rate squared during inflation. This means that the field ψ providing the energy density dominating during the inflationary stage is well fixed at the false ground state. The false vacuum can decay to the real vacuum at $\psi = \pi$ by a tunneling effect but the necessary time, computed in [14], is exponentially large. Without any interference from outside, inflation will last almost indefinitely.

2.1 Anti-$D3$-branes in the throat

Inflation may stop if extra anti-$D3$-branes are sent in and increase the value of p. The crucial point is that there is a maximal value p_{cr} of p/M for which the potential $V(\psi)$ has a false vacuum. For $p > p_{cr}$, the potential is a monotonic decreasing function of ψ (see figure 3).

If we send in a sufficient number of anti-$D3$-branes and p reaches the critical value the false vacuum disappears and ψ starts rolling down to the real vacuum at $\psi = 0$ finishing the inflationary period.

We suppose that the extra anti-branes were originally present in the compactification at $r > r_{UV}$ and that they left the UV region of the compactification for dynamical reasons. Their initial energy at $r = r_{UV}$ is obtained by multiplying the brane tension with the red-shift factor. For a single anti-brane entering the throat, the initial energy is $T_3(\frac{r_{UV}}{R})^4$. We require that the anti-$D3$-brane is a small perturbation of the system and the vacuum energy is still determined by the IR stack of branes. To this purpose, we must require

$$T_3\left(\frac{r_{UV}}{R}\right)^4 \lesssim 2p\,T_3\left(\frac{r_0}{R}\right)^4. \tag{2.8}$$

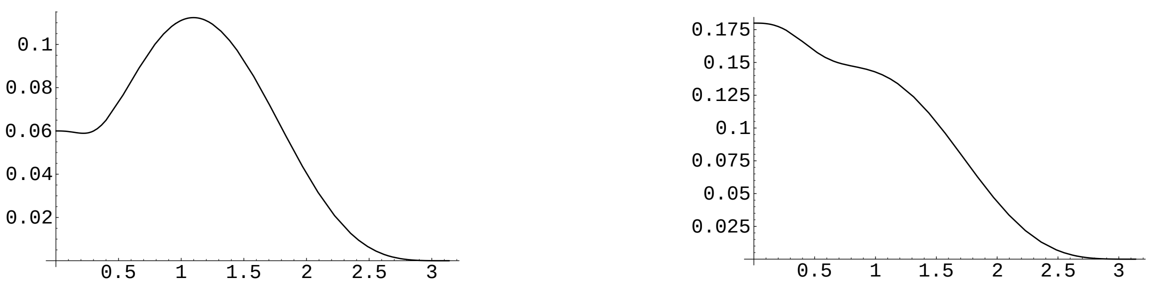

Figure 3: The function $V_2(\psi) - \frac{1}{2\pi}(2\psi - \sin 2\psi) + \frac{p}{M}$, equal to the potential $V(\psi)$ up to an overall scale, for $p/M = 0.03$, where there is a false vacuum, and for $p/M = 0.09$ where the potential is monotonic.

To satisfy this condition, we need to consider a mild warping in the throat or large values of the fluxes. Both requirements can be obtained by varying the integers M, K (and the internal manifold) while keeping order unity values for the fundamental parameter g_s and $\alpha' \gg M_p^{-2}$, $T_3 \lesssim 1/\alpha'^2 \ll M_p^4$. For instance, a judicious choice would be $K \sim M \gg p \gtrsim (R/r_0)^4$ and therefore $N \sim M^2$. If so, $H_*^2 \sim T_3/M_p^2$ and $m_\psi^2/H_*^2 \gg 1$ when $\alpha' M_p^2 \gg \sqrt{p}/(8\pi^3 g_s)$.

Once in the throat, the anti-$D3$-brane feels a force toward the IR that can be estimated as follows. In flat space, there is no-force between anti-$D3$-branes since they are mutually BPS. In the curved background, however, there is a force due to gravity and the RR forms. The effective lagrangian for the radial position of the brane is computed in appendix B and reads

$$L(r) = -T_3 \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} (\partial_\mu r)(\partial_\nu r) + \frac{\mathcal{R}}{12} r^2 - 2h^{-1}(r) \right). \tag{2.9}$$

In this equation the potential is twice the red-shift factor, since the contribution of the RR forms is equal to that of gravity. The computation leading to equation (2.9) is similar to that performed in [6], where a $D3$-brane was moving in the throat. For example, there is the same coupling to the Ricci scalar. A crucial difference with [6] is that, in their case, the potential for a $D3$-brane was flat and slowly varying. An anti-$D3$-brane has instead a large potential $V \sim r^4/R^4$ and is rapidly attracted to the IR. The lagrangian (2.9) is valid for $r \gg r_0$. Once the anti-brane reaches $r = r_0$, one should analyze the interaction between anti-$D3$-branes more closely. In first approximation, the net effect of sending in a single anti-$D3$-brane is to shift $p \to p+1$.

2.2 Volume stabilization

We suppose that all the moduli of the compactification have been stabilized. Backgrounds like the KS one are particularly appealing because all the complex structure moduli of the internal manifold are stabilized by the fluxes [10]. To this purpose, we can also turn on extra fluxes along the other cycles of the internal manifold. One Kähler modulus, the internal volume, however, is left massless. This is a typical problem in all string compactifications. One can imagine, as in [13], that a non-perturbative superpotential is generated for the volume. Non-perturbative potentials can be generated, for example, by the gaugino condensation in gauge groups arising from branes in the UV region (see figure 1) [13]. These non-perturbative effects arising from the the UV will not affect the dynamics of the stack of anti-branes that are located in the IR region. Moreover, $D7$ branes are naturally present in F-theory compactifications that could serve as a compactification of the KS solution [10]. Each strongly coupled gauge factor $\mathrm{U}(N_c)$ would produce a superpotential for the volume of the form

$$W \sim e^{-2\pi\rho/N_c}, \tag{2.10}$$

where $\rho = \frac{R_{CY}^4}{\alpha'^2 g_s} + i\sigma$, R_{CY} being the radius of the internal manifold, is a chiral superfield whose real part is related to the internal volume and whose imaginary part is an axion-like field. Formula (2.10) follows from the fact that a $D7$-branes wrapped on four internal directions has a gauge coupling $1/g_{YM}^2 \sim R_{CY}^4/g_s$. We can even suppose that multiple sets of $D7$ branes undergo gaugino condensation. In this case we can easily get racetrack potentials consisting of multiple exponentials where the volume is stabilized with a large mass while the axion is much lighter, as we will discuss in the next section. Under these circumstances, the axion will play the role of the curvaton.

Notice that both the wandering and the IR anti-D3-branes generates an extra contribution to the potential for the volume. For example, the IR stack of branes gives a contribution $\sim \frac{1}{(\rho+\bar{\rho})^2}$ [13, 6].[1] Similarly to what supposed in [6], we will assume that the scales in the superpotentials are such that the volume stabilization is not affected by the contributions present during inflation so that the volume is frozen to its minimum.

As pointed out in [6], there is the extra problem that the stabilization of the volume induces a mass term for the world-volume brane scalar fields of the order of the Hubble constant. An explicit coupling to the Ricci tensor has been included in the lagrangian (2.9) for the scalar r; it does not affect our arguments as we will discuss in the next section.

3. Inflation and the cosmological perturbations

As we have pointed out in the previous section, the primordial stage of inflation is driven by the vacuum energy density (2.6) stored in the false vacuum provided by a set of p anti-$D3$-branes sitting in the deep IR region of the metric. In terms of the four-dimensional effective description, the inflaton is identified with a four-dimensional scalar field parameterizing the angular position ψ of the anti-$D3$-branes in the internal directions. The curvature of the potential around the minimum is much larger than the Hubble rate (2.7) during inflation. Since slow-roll conditions are not attained, the curvature perturbation associated to the quantum fluctuations of the inflaton field ψ is heavily suppressed. Its amplitude goes like $e^{-m_\psi^2/H_*^2}$, with $m_\psi \gg H_*$, and the spectrum in momentum space is highly tilted towards the blue [17].

In our set-up inflation is stopped by whatever mechanism increases the number of p anti-$D3$-branes beyond some critical value. Indeed, as we have explained in the previous section, the false vacuum for the potential exists only if the number of anti-$D3$-branes is smaller than a critical number p_{cr}. If the number p changes by an amount Δp (typically a fraction of M) becoming larger than a critical value, the curvature around the false vacuum becomes negative, the system rolls down the supersymmetric vacuum and the graceful exit from inflation is attained. This mechanism is different from what happens in the four-dimensional hybrid model of inflation [18] where inflation is ended by a water-fall transition triggered by the same scalar field responsible for the cosmological perturbations and is more reminiscent of the recently proposed idea of old new inflation described in ref. [19].

We may envisage therefore the following situation. Suppose that a number of anti-$D3$-brane is left in the UV region of the compactification for dynamical reasons. Once in the throat, each anti-$D3$-brane feels an attractive force toward the IR proportional to $(r/R)^4$, where r stands for the modulus parameterizing the distance between each anti-$D3$-brane and the IR region. The dynamics of such a modulus is described in terms of

[1] This behavior can be understood by a Weyl rescaling $g \to g/\mathcal{V}$ (in order to decouple metric and volume fluctuations [10]) and by including the warp factor dependence on the volume [6].

the lagrangian (2.9). Notice, in particular, that the canonically normalized field $\phi = \sqrt{T_3}\, r$ during the inflationary stage receives a contribution to its mass squared proportional to H_*^2,

$$\Delta m_\phi^2 = -\frac{\mathcal{R}}{6} = 2H_*^2\,, \tag{3.1}$$

where we have made of use of the fact that during a de Sitter phase $\mathcal{R} = -12H_*^2$. This contribution spoils the flatness of the potential in slow-roll stringy models of inflation where the inflaton field is identified with the inter-distance between branes [6]. In our case, however, such a contribution is not dangerous and its only effect is to suppress the quantum fluctuations of the field ϕ.

Once an anti-$D3$-brane appears in the UV region, it rapidly flows towards the IR region under the action of a quartic potential $\lambda\phi^4$, where $\lambda \sim (T_3 R)^{-1}$, and it starts oscillating around the value $\phi_0 = \sqrt{T_3}\, r_0$ under the action of the quadratic potential $\sim \Delta m_\phi^2 \phi^2$. Since the Universe is in a de Sitter phase, the amplitude of the oscillations decreases as

$$\phi = \phi_i\, e^{-\frac{3}{2}(\mathcal{N}-\mathcal{N}_i)}\,, \tag{3.2}$$

where $\mathcal{N} = \ln(a/a_i)$ is the number of e-foldings and the subscript i denotes some initial condition. Once the energy density stored in the oscillations becomes smaller than $\sim 1/\alpha'^2$ the anti-$D3$-brane stops its motion and gets glued with the p anti-$D3$-branes in the IR. At this stage, their number is increased by one unity, going from p to $p+1$. Once the number of anti-$D3$-brane becomes equal to p_{cr}, inflation ends since the system rolls down towards the supersymmetric vacuum at $\psi = \pi$. At this stage the vacuum energy is released. From the four-dimensional point of view, this reheating process takes place through the oscillations of the inflaton field ψ about the minimum of its potential with mass squared $\sim (1/\alpha')(r_0/R)^2$. From the higher-dimensional point of view, the reheating process corresponds to a transition between a metastable string configuration with fluxes M, K and anti-branes to a stable one. The anti-branes annihilate releasing energy and changing the values of the fluxes, $M \to M-p, K \to K-1$. There is also a complementary description of the reheating process in terms of the holographic dual. The IR description of the original system can be given using the dual gauge theory $\mathrm{SU}(2M-p)\otimes\mathrm{SU}(M-p)$ (this is the endpoint of the KS cascade [9, 14]). The reheating corresponds to the transition from a metastable non-supersymmetric baryonic vacuum to the the supersymmetric one [14]. The details of reheating on other three or seven branes supporting our Universe are worth studying in more detail, but this goes beyond the scope of this paper.

We could try to obtain more realistic IR physics by introducing other three or seven branes, which might support our universe. An attempt to embed the Standard Model of particle interactions in the set-up described in this paper was recently done in ref. [12]

3.1 The number of e-foldings

In our set-up, the total number of e-foldings depends on the initial number of p anti-$D3$-branes sitting in the deep IR region (the only necessary condition is $p < p_{cr}$), on the number of wandering anti-$D3$-branes in the bulk and, also, on the time interval separating each wandering anti-$D3$-brane from the next one.

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Due to our ignorance on the initial state, one can imagine some extreme situations. For instance, suppose that the initial number of p anti-$D3$-branes sitting in the deep IR region differs from p_{cr} only by one unity. One wandering anti-$D3$-brane is therefore enough to stop inflation. Using eq. (3.2) and taking as ϕ_i the (conservative) value at which the quadratic potential dominates over the quartic one, the number of e-foldings corresponding to the motion of a single anti-$D3$-brane before capturing is

$$\mathcal{N} \sim \frac{2}{3} \ln \left[\frac{pN\,(r_0/R)^4}{M_p^2 \alpha'} \right]. \tag{3.3}$$

One has to impose pN to be much larger than the warping factor $(R/r_0)^4$ in order to get a sizable number of e-foldings. This means that the minimum number ~ 50 of e-foldings necessary to explain the homogeneity and isotropy of our observed Universe cannot be explained in terms of a single wandering anti-$D3$-brane, but is likely to be provided by the prolonged de Sitter phase preceding the appearance of the wandering anti-$D3$-brane. In such case, we need to assume that the antibrane spends a large time in the ultraviolet region before being attracted in the region of the potential where the latter is quartic. This time depends on the initial condition of the anti-D3-brane and on the dynamics that causes its detachment from the UV region. For instance, in the far ultraviolet region, the antibranes might be stuck to a set of D7 branes in a sort of metastable state and can be realesed with a nonvanishing probability.

As an alternative, consider the case in which the initial number of p anti-$D3$-branes sitting in the deep IR region differs from p_{cr} by several units, say $\sim M$. Under these circumstances several wandering anti-$D3$-branes are needed to exit from inflation. Supposing that the wandering anti-$D3$-branes are well separated in time, the total number of e-foldings between the appearance of the first anti-$D3$-brane and the end of inflation is given at least as large as

$$\mathcal{N} \sim \frac{2}{3}\, M \ln \left[\frac{pN\,(r_0/R)^4}{M_p^2 \alpha'} \right]. \tag{3.4}$$

We conclude that the last 50 e-foldings before the end of inflation might well correspond to the period during which $M \sim 50$ anti-$D3$-branes flow into the throat.

3.2 The generation of the cosmological perturbations

As we have pointed out in several occasions, both the inflaton and the modulus parameterizing the distance between the wandering anti-$D3$-branes and the IR region are four-dimensional degrees of freedom whose mass is much larger than the Hubble rate during inflation. This implies that their quantum fluctuations are not excited during inflation. Fortunately, it has recently become clear that the curvature adiabatic perturbations responsible for the structures of the observed Universe may well be generated through the quantum fluctuations of some field other than the inflaton [16]. The curvaton scenario relies on the fact that the quantum fluctuations of any scalar field in a quasi de Sitter epoch have a flat spectrum as long as the mass of the field is lighter than the Hubble rate. These

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fluctuations are of isocurvature nature if the energy density of the scalar field is subdominant. The scalar field, dubbed the curvaton, oscillates during some radiation-dominated era, causing its energy density to grow and thereby generating the curvature perturbation.

The requirement that the effective curvaton mass be much less than the Hubble parameter during inflation is a severe constraint. In this respect the situation for the curvaton is the same as that for the inflaton in the inflaton scenario. To keep the effective mass of the inflaton or curvaton small enough, it seems natural to invoke supersymmetry and to take advantage of one of the many flat directions present in supersymmetric models. However, one has to check no effective mass-squared $\sim H_*^2$ is generated during inflation.

An alternative possibility for keeping the effective mass sufficiently small is to make the curvaton a pseudo Nambu-Goldstone boson (PNGB), so that its potential vanishes in the limit where the corresponding global symmetry is unbroken. Then the effective mass-squared of the curvaton vanishes in the limit of unbroken symmetry and can indeed be kept small by keeping the breaking sufficiently small. The curvaton as a PNGB has been studied in detail in ref. [20].

As we have anticipated in section 2, non-perturbative superpotentials are expected to be generated for the volume modulus. They can be generated, for example, by the gaugino condensation in gauge groups arising from branes in the UV region and wrapped on four internal directions. This happens for $D7$ branes which are naturally present in F-theory compactifications that could serve as a compactification of the KS solution [10]. If multiple sets of $D7$ branes undergo gaugino condensation, one can get racetrack potentials consisting of multiple exponentials. If we suppose that the Kähler potential does not depend upon the imaginary part of the volume modulus ρ, let us call it $\sigma = \mathrm{Im}\,\rho$, and if the non-perturbative superpotential is of the form

$$W \sim e^{-a\rho} + e^{-b\rho} + \cdots \tag{3.5}$$

where $a < b$ are some positive constants, then the axion-like field σ receives a mass which is suppressed by the exponential $\sim e^{-(b-a)\,\mathrm{Re}\,\rho}$ with respect to the mass of the volume $m_{\mathcal{V}}$ [21]

$$m_\sigma^2 \sim e^{-(b-a)\,\mathrm{Re}\,\rho}\, m_{\mathcal{V}}^2\,. \tag{3.6}$$

Because of the exponential suppression, the condition $m_\sigma^2 \ll H_*^2$ during inflation does not require any particular fine-tuning. The axion σ plays the role of the curvaton. Furthermore, since the non-perturbative superpotentials arise in the UV region, no warping suppression is expected and the axion scale f will be of the order of M_p in the four-dimensional effective theory. The condition $m_\sigma/f \ll 10^{-2}$ imposed in order to be sure that inflation lasts enough for the curvaton to be in the quantum regime [20] is likely to be satisfied.

The requirement that the curvaton potential be negligible during inflation corresponds to

$$f m_\sigma \ll M_p H_*\,. \tag{3.7}$$

Since it is assumed that the curvaton is light during inflation, $m_\sigma \ll H_*$, on super-horizon scales the curvaton has a classical perturbation with an almost flat spectrum given by

$$\langle \delta\sigma^2 \rangle^{\frac{1}{2}} = \frac{H_*}{2\pi}\,. \tag{3.8}$$

When, after inflation, $H \sim m_\sigma$, the field starts to oscillate around zero. At this stage the curvaton energy density is $\rho_\sigma = \frac{1}{2} m_\sigma^2 \sigma_*^2$ while the total is $\rho \sim H^2 M_p^2$. Here σ_* is the value of the curvaton during inflation. The fraction of energy stored in the curvaton is therefore $\sim (\sigma_*/M_p)^2$, which is small provided that $\sigma_* \ll M_p$.

After a few Hubble times the oscillation will be sinusoidal except for the Hubble damping. The energy density ρ_σ will then be proportional to the square of the oscillation amplitude, and will scale like the inverse of the locally-defined comoving volume corresponding to matter domination. On the spatially flat slicing, corresponding to uniform local expansion, its perturbation has a constant value

$$\frac{\delta\rho_\sigma}{\rho_\sigma} = 2q \left(\frac{\delta\sigma}{\sigma}\right)_* . \tag{3.9}$$

The factor q accounts for the evolution of the field from the time that m_σ/H becomes significant, and will be close to 1 provided that σ_* is not too close to the maximum value πv. The curvature perturbation ζ is supposed to be negligible when the curvaton starts to oscillate, growing during some radiation-dominated era when $\rho_\sigma/\rho \propto a$. After the curvaton decays ζ) becomes constant. In the approximation that the curvaton decays instantly (and setting $q = 1$) it is then given by [16]

$$\zeta \simeq \frac{2\gamma}{3} \left(\frac{\delta\sigma}{\sigma}\right)_* , \tag{3.10}$$

where

$$\gamma \equiv \left.\frac{\rho_\sigma}{\rho}\right|_D , \tag{3.11}$$

and the subscript D denotes the epoch of decay. The corresponding spectrum is

$$\mathcal{P}_(\zeta)^{\frac{1}{2}} \simeq \frac{2\gamma}{3} \left(\frac{H_*}{2\pi\sigma_*}\right) . \tag{3.12}$$

It must match the observed value 5×10^{-5} [22] which means that $H_*/2\pi\sigma_* \simeq 5 \times 10^{-4}/\gamma$. The current WMAP bound on non-gaussianity [23] requires $\gamma \gtrsim 9 \times 10^{-3}$. In terms of the fundamental parameter of our theory, we get

$$\frac{p\,T_3}{M_p^4} \left(\frac{r_0}{R}\right)^4 \sim \frac{10^{-6}}{\gamma^2} \lesssim 10^{-2} , \tag{3.13}$$

where we have taken $\sigma_* \sim f \sim M_p$.

Before closing this section, we would like to mention a possible alternative for the curvaton field. During its motion toward the IR, the anti-$D3$-brane can fluctuate in the internal angular directions. The scalar fields associated with the angular positions are almost massless. In particular, being angles, they do not get masses from the volume stabilization mechanism. In the Kähler potential of four-dimensional supergravity the volume modulus ρ always appears in the combination $\rho + \bar{\rho} - k(\phi_i, \bar{\phi}_i)$ [24], where ϕ_i collectively denote the position of the branes in the six internal directions, and k is the Kähler potential for the geometry. This coupling generates a mass for the fluctuations ϕ_i. However, at least for large values of r, the geometry has several isometries and $k(\phi, \bar{\phi})$ does not depend on some of the angles. For example, for large r the geometry of the

KS throat is that of a cone over the Einstein manifold $T^{1,1} = \mathrm{SU}(2) \otimes \mathrm{SU}(2)/\,\mathrm{U}(1)$. The internal geometry has therefore the isometry $\mathrm{SU}(2) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ that guarantees the independence of $k(\phi, \bar{\phi})$ from some angles. In this way, the form of potential for the angular fluctuations is not affected by the stabilization mechanism and leaves some angles much lighter than the Hubble rate during inflation and they may play the role of the curvaton(s). The isometries do disappear in the IR region, since there the singular cone over $T^{1,1}$ is made smooth by deforming the tip of the cone. One then expects that such curvatons becomes massive after inflation and decay during or after reheating. If so, one expects $\gamma \lesssim 1$ thus enhancing the non-gaussian signature [25].

Another relevant prediction of our model is that the Hubble rate during inflation is not necessarily small. This is different from what usually assumed in models which make use of the curvaton mechanism to produce cosmological perturbations where the Hubble rate is tiny in order to suppress the curvature perturbations from the inflaton field. In our set-up the latter are suppressed on superhorizon scales not by the smallness of H_*, but by the fact that the inflaton field ψ is not light during inflation. Therefore, a generic prediction of our model is that gravitational waves may be produced at an observable level and close to the present bound corresponding to $H_* \lesssim 10^{14}\,\mathrm{GeV}$ [26].

4. Conclusions

Brane-world scenarios in string theory offer new ways of obtaining a primordial period of inflation necessary to account for the homogeneity and isotropy of our observed Universe. At the same time, they pose some challenges. In warped geometries, implementing volume stabilization spoils the flatness of the inflaton potential in brane-antibrane inflation [6] unless a shift symmetry is preserved [7, 8].

In this paper we have described a specific example of string compatification with warped metric which leads to the old inflation scenario and does not require any slow-roll inflaton potential. Warping is introduced by antisymmetric forms with nonvanishing fluxes in the internal directions of the compactification. A stack of anti-$D3$-branes reside in the deep IR region of the warped metric and, if their number is not larger than a critical value, supersymmetry is broken and a false vacuum is formed. From the four-dimensional point of view, the system is described in terms of a scalar field parameterizing the position of the antibranes along the internal directions. Such a scalar is well anchored at the false vacuum with a mass much larger than the Hubble rate during inflation. Slow-roll conditions are violated. Graceful exit from inflation is attained augmenting the number of antibranes in the IR region by sending antibranes towards the IR from the UV region of the warped geometry. These single wandering antibranes are inevitably attracted by the stack of antibranes in the IR and, after a few oscillations, end up increasing the number of the antibranes in the stack thus stopping inflation. Cosmological curvature perturbations are generated through the curvaton mechanism. We have primarily focused on the imaginary part of the volume modulus as a curvaton field, showing that its mass can be easily lighter than the Hubble rate during inflation. The curvaton acts as a PNGB whose dynamics has been thoroughly studied in [20].

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There are interesting issues which would deserve further and careful investigation. First of all, we have not studied in this paper the process of reheating. From the four-dimensional point of view, reheating takes place through the oscillations of the inflaton field ψ about the minimum of its potential with mass squared $\sim (1/\alpha')(r_0/R)^2$. From the higher-dimensional point of view, reheating corresponds to the disappearance of the stack of antibranes and the appearance of fluxes. The final state is supersymmetric and of the same form of the KS solution with a small change in the fluxes: $M \to M - p$ and in $K \to K - 1$. Using the holographic duality, we can identify the final state with an $\mathrm{SU}(2M - p) \otimes \mathrm{SU}(M - p)$ supersymmetric gauge theory. We could try to obtain more realistic IR physics by introducing other three or seven branes, which might support our universe. An attempt to embed the Standard Model of particle interactions in the set-up described in this paper was recently done in ref. [12]. With a specific model at hand the details of the transition to the final state and the reheating process could be studied. It would be also interesting to see whether this transition leaves behind topological defects [27] and which is, eventually, their impact of the subsequent cosmological evolution. We will come back to all these issues in the next future.

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A. The KS solution

Warped solutions with a throat of the form (2.1) are common in string theory. They can be generated by either a stack of branes or by using solutions with RR fluxes. The two pictures (branes versus fluxes) are dual to each other in the sense of the AdS/CFT correspondence. We will consider solutions with fluxes. One can take many examples out of the AdS/CFT literature. In this context one usually consider non-compact solutions with a radial coordinate r. To obtain a compact model, one must truncate the metric at a certain UV scale r_{UV} and glue a compact manifold for $r > r_{UV}$. Varying the internal manifold and the combinations of brane/fluxes, one can engineer various supersymmetries.

For example, if we choose $h(r) = R^4/r^4$ and the metric for the round five-sphere for $ds_{(5)}$, we obtain

$$ds^2 = \frac{r^2}{R^2} dx_\mu dx^\mu + \frac{R^2}{r^2} dr^2 + R^2 ds_{S_5}\,, \tag{A.1}$$

the product of $AdS_5 \times S^5$. The solution also contains N units of flux for the RR four-form $C_{(4)}$ along S^5. This choice of warp factor corresponds to a maximally supersymmetric solution of string theory and it is equivalent to the RSII model. The compact manifold glued for $r > r_{UV}$ corresponds to an explicit realization of the Plank brane of the RS scenario. The RSI model can be obtained by truncating the metric (2.1) at $r = r_0$ by the insertion of an IR brane. In contrast to the RSII model, the warp factor is now bounded above zero and has a minimal value that has been used to study the hierarchy problem.

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The IR brane can be replaced by any regular geometry that has a non-zero minimal warp factor. A regular type-IIB solution with background fluxes with this property has been found by Klebanov and Strassler. In terms of an appropriate radial coordinate τ for which the IR corresponds to $\tau = 0$, the KS solution has a the form

$$ds^2 = h^{-1/2}(\tau) dx_\mu dx^\mu + h^{1/2}(\tau) ds^2_{(6)}\,, \tag{A.2}$$

with

$$\begin{aligned} h(\tau) &= \text{const} \times I(\tau)\,, \\ I(\tau) &= \int_\tau^\infty \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3} \end{aligned} \tag{A.3}$$

and with a complicated internal metric, which depends on τ and five angles. Here $ds_{(6)}$ is the metric for a deformed conifold. It is obtained by taking a cone over a five-dimensional Einstein manifold ($T^{1,1}$) with the topology of $S^3 \times S^2$ and by deforming the tip of the cone in order to have a smooth manifold. The resulting manifold has a non-trivial S^3 cycle. In addition to the non-trivial metric, there are fluxes for the antisymmetric forms of type-IIB supergravity. The solution preserves $N = 1$ supersymmetry.

The non-compact KS solution can be embedded in a a genuine string compactification as explained in [10]. The most convenient way is to consider F-theory solutions that can develop a local conifold singularity. An explicit example is provided in [10]. In the compact solution, the R-R and NS-NS two-forms have integer fluxes along the S^3 cycle of the conifold (call it A) and along its Poincaré dual B, respectively:

$$\frac{1}{(2\pi)^2\alpha'} \int_A F = M\,, \qquad \frac{1}{(2\pi)^2\alpha'} \int_B H = -K\,, \tag{A.4}$$

where F and H are the curvatures of $C_{(2)}$ and $B_{(2)}$. In order to avoid large curvature in the solution that would invalidate the supergravity approximation, the integers M and K must be large. The solution has a minimal warp factor that is given by $e^{-\frac{2\pi K}{3 g_s M}}$.

We can include wandering $D3$ and anti-$D3$-branes in the compactification. For this we must ensure that the total $D3$-charge is zero, as required by Gauss law in the case of a compact manifold. The effective $D3$-charge gets contribution from $D3$ and anti-$D3$-branes and from the various couplings of $C_{(4)}$ to the two-forms and to the curvature of the internal manifold. The resulting constraint is

$$\frac{\chi}{24} = N_3 - \bar{N}_3 + KM\,, \tag{A.5}$$

where $N_3, \bar{N}_3$ are the number of branes and anti-branes and χ is the Euler characteristic of the manifold used for the F-theory compactification. In all our examples, $N_3 = 0$ and the number of anti-branes p is much smaller than the background flux M. Defining $N = \chi/24$ the effective $D3$-charge, we have to satisfy $N = KM$.

We will only need the asymptotic behavior of the metric (A.2) for large and small radial coordinate, where it assumes the form given in (2.1). For large τ, it is convenient to use the variable $r^2 \sim e^{2\tau/3}$ and we have

$$h(r) = \frac{R^4}{r^4} \left(1 + \text{const } \log \frac{r}{r_{cr}}\right) \tag{A.6}$$

which corresponds to a logarithmic deformation of AdS. For small τ, $h(\tau)$ approaches a constant and the internal metric $ds_{(6)}$ is the product $R^3 \times S^3$. The radius square of S^3 (measured in ten dimensional units) is of order $g_s M\alpha'$ and this quantity must be large for the validity of the supergravity approximation.

According to the holographic interpretation of the RS model, the IR part of the geometry corresponds to four-dimensional matter fields that are determined by using the AdS/CFT correspondence: the IR part of the metric in the RSII model corresponds to a CFT theory, the IR part of the metric of the KS solution corresponds to a pure SYM theory. In particular, the holographic dual of the KS solution, for $p = 0$ and N multiple of M ($N = MK$), is a SU(N+M)$\otimes$SU(N) gauge theory, which undergoes a series of Seiberg duality leaving a pure confining SU(M) SYM theory in the IR [9]. For $p \neq 0$, we can expect that the anti-D3-branes annihilate p physical branes giving a SU($N+M-p$)$\otimes$SU($N-p$) gauge theory. As discusses in [9, 14], the Seiberg duality cascade stops at SU($2M-p$)$\otimes$SU($M-p$), a gauge theory with still a moduli space of vacua.

B. The wandering anti-$D3$-branes

An anti-$D3$-brane entering the throat region at $r \gg r_0$ can be considered as a probe. It will feel a force toward the IR region. Anti-$D3$-branes are mutually BPS and, therefore, in first approximation, the contribution to the force from the IR stack of p anti-branes can be neglected. The wandering anti-brane will feel a potential due to the non-trivial warp factor and the RR-fields background. We also suppose that the stack of IR branes is not modifying the background and it has the only effect to induce a non-zero vacuum energy. In a more precise calculation, one should consider a de-Sitter deformed KS solution [28].

In the probe approximation, the back-reaction on the metric can be neglected. The fields in the world-volume effective action couple to the metric and to all background antisymmetric forms. Our background for $r \gg r_0$ has the form given in (2.1) and a non-vanishing four-form

$$C_{(4)} \sim h^{-1}(r)\epsilon_{0123}\,. \tag{B.1}$$

We write the effective action for the position of a $D3$ or anti-$D3$-brane probe,

$$S = -T_3 \int d^4\sqrt{g_{IND}} + qT_3 \int C_{(4)}\,. \tag{B.2}$$

The first term in this equation is the Born-Infeld action that depends on the induced metric and the second term is the Wess-Zumino coupling to the RR fields. Here $T_3 = \frac{1}{(2\pi)^3(\alpha')^2 g_s}$ is the tension of the brane and q is the charge under $C_{(4)}$ which is +1 for $D3$-branes and -1 for anti-$D3$-branes. With the given values for the background fields,

$$S = -T_3 \int d^4x h^{-1}(r)\sqrt{1 - h(r)(\partial r)^2} + qT_3 \int d^4 h^{-1}(r)\,. \tag{B.3}$$

We see that the potential energy for a brane is given by two contributions, one coming from the non-trivial red-shift of the metric and a second one coming from the RR fields. A $D3$-branes feels no force in this background

$$V(r) = -T_3 h^{-1}(r) + T_3 h^{-1}(r) = 0\,. \tag{B.4}$$

This fact can be understood easily if one invoke the duality that relates our background with fluxes to systems of $D3$-branes. The KS solution is dual to an $N = 1$ gauge theory with a moduli space of vacua. In the language of branes, this corresponds to the possibility to separate one or more $D3$-branes from the stack with no cost in energy. The gravitational attraction between branes is compensated by the charge repulsion due to the RR fields. The fact that a $D3$-brane is a BPS object in the background was used in [6] to obtain an almost flat potential for the moving brane.

On the other hand, the anti-$D3$-branes will feel a potential

$$V(r) = -T_3 h^{-1}(r) - T_3 h^{-1}(r) = -2T_3 h^{-1}(r)\,. \tag{B.5}$$

In this case, indeed, the charge of the anti-$D3$-branes has changed sign and the Coulomb and gravitational forces will add.

In the lagrangian for the anti-$D3$-branes we should also include a coupling to the four-dimensional curvature $\mathcal{R}$. To our purpose, we can approximate the metric for large r with an AdS metric. In a five-dimensional AdS background this coupling has been computed in [29] for large r. The coupling is generated by the Born-Infeld part of the action and therefore has the same sign for both $D3$ and anti-$D3$-branes. The result is that of a conformally coupled scalar.

Including all contributions the effective action for the anti-$D3$-branes reads (up to two derivatives)

$$L(r) = -T_3 \int d^4x\sqrt{g}\left(\frac{1}{2}g^{\mu\nu}(\partial_\mu r)(\partial_\nu r) + \frac{\mathcal{R}}{12}r^2 - 2h^{-1}(r)\right). \tag{B.6}$$

C. The IR anti-branes

Our purpose in this section is to explain formula (2.3)

$$L(\psi) = -T_3 \int d^4x\sqrt{g}\frac{r_0^4}{R^4}\left[M\left(V_2(\psi)\sqrt{1-\frac{1}{r_0^2}(\partial\psi)^2} - \frac{1}{2\pi}(2\psi - \sin 2\psi)\right) + p\right], \tag{C.1}$$

giving a brief account of how it is obtained. Details can be found in [14]. In this section we put $\alpha' = 1$.

In the IR the internal metric is the product $R^3 \times S^3$. The total geometry is $R^7 \times S^3$ with the radius square of S^3 being $b_0^2 g_s M\alpha'$ ($b_0^2 \sim 0.9$). As already mentioned, this quantity must be large for the validity of the supergravity approximation. We can choose the metric for S^3 as

$$b_0^2 g_s M(d\psi^2 + \sin^2\psi d\Omega_{(2)})\,. \tag{C.2}$$

There are also non-trivial RR and NS-NS form. From the condition $\int_{S^3} F = 4\pi^2 M$, one easily gets the potential

$$C_{(2)} = 4\pi M\left(\psi - \frac{\sin 2\psi}{2}\right) d\Omega_{(2)} \tag{C.3}$$

With a more accurate computation using the equations of motion one can also determine $H_{(3)}$ and its dual $H_{(7)} = *H_{(3)}$ ($H_{(7)} = dB_{(6)}$) [14].

The crucial observation for studying the dynamics of the system is that a NS-brane wrapped on a two sphere in S^3 with p units of world-volume flux has the same quantum number of the stack of p anti-branes. This is a standard observation in string theory (for a partial list of references related to our configuration see [30]). Consider indeed a NS-brane wrapping the cycle specified by the angle ψ. It has a world-volume action

$$S = -\frac{\mu_5}{g_s^2}\int d^6x\sqrt{G_{IND} + g_s(2\pi F_{(2)} - C_{(2)})} + \mu_5\int d^6x B_{(6)}\,. \tag{C.4}$$

This action can be obtained by S-duality from the Born-Infeld and Wess-Zumino action for D5 branes. In string units $\alpha' = 1$, $\mu_5 = \frac{1}{(2\pi)^5} = \frac{\mu_3}{4\pi^2}$ is the tension of a $D5$-brane. $F_{(2)}$ is the U(1) world-volume field of the brane that, for gauge invariance, must always couple to $C_{(2)}$. Beside the Wess-Zumino term $B_{(6)}$, corresponding to the unit charge of the NS brane, there is a Wess-Zumino coupling $C_{(4)} \wedge (2\pi F_{(2)} - C_{(2)})$. This term is responsible for the induced anti-$D3$-charge when we introduce a non-zero flux for $F_{(2)}$ on the two-sphere,

$$\int_{S^2} F_{(2)} = 2\pi p\,. \tag{C.5}$$

When the NS brane is at $\psi = 0$, the two-sphere is vanishing and the five-brane becomes effectively a three brane. This three brane is not tensionless because $F_{(2)}$ enters explicitly in the Born-Infeld action. By reducing the action (C.4) on the vanishing two-sphere we obtain a three brane with tension $4p\pi^2 T_5 = pT_3$ and negative $D3$-charge -p. These are the quantum numbers of a stack of p anti-$D3$-branes.

The description in terms of a wrapped NS brane it useful when $\psi \neq 0$ and it shows that the stack of branes can lower its energy by expanding into an NS brane. Formula (C.1) is obtained from (C.4) by integrating on the two-sphere. The potential

$$V_2(\psi) = \frac{1}{\pi}\sqrt{b_0^2 \sin^4\psi + \left(\frac{\pi p}{M} - \psi + \frac{\sin 2\psi}{2}\right)^2} \tag{C.6}$$

is the contribution of the determinant of the two by two matrix $G_{IND} + g_s(2\pi F_{(2)} - C_{(2)})$ in the directions of S^2. The contribution $\sim 2\psi - \sin 2\psi$ to the potential comes from the integral of $B_{(6)}$. There is no contribution from $C_{(4)}$ since the four-form vanishes in the IR [9]. Finally the contribution of p units of $D3$ tension to (C.1) comes from the background fluxes via the tadpole cancellation condition [14]. Finally, the potential (C.1) is obtained by introducing the appropriate warp factor everywhere. The total potential in ψ has a form that depends on p and it is pictured in figure 3. The real minimum is at $\psi = \pi$ and it has vanishing energy.

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